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Chapter 7

LINEAR PROGRAMMING FOR DAIRY HERD SIMULATION AND OPTIMIZATION: AN INTEGRATED APPROACH FOR DECISION-MAKING

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ABSTRACT

The use of linear programming (LP) in farming systems is not a new concept. Linear programming has been used extensively to suggest the impact of alternative management practices at the whole farm level. Although these applications included livestock practices, there have not been many studies that formally and systematically investigated dairy herd systems. Linear programming can be a powerful tool to simulate and optimize the dairy herd system inside a Markov-chain structure. On the other hand, the concept of dynamic programming (DP) for a dairy herd has long been recognized and used to find optimal policies for dairy herd management. Various options have been analyzed to find optimal

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replacement policies, reproductive parameters, and feeding strategies in dairy herds by using value or policy iteration methods. However, even though the formulation has been available since the 1980s, the solution of DP using LP has not been widely explored probably because the computer and software systems did not support the solution of real and practical problems. The formulation of DP as an LP problem for real, but large problems is now feasible and has substantial advantages over other methods because it allows the inclusion of the interaction of herd mates, solving for sub-optimal conditions, controlling efficiently for the time steps of the analysis, and uses standard LP algorithms for solution.

In the present chapter we discuss the application of LP in dairy herd management to solve DP problems and to propose stochastic simulation and optimization in a Markov-chain structure for decision-making in modern dairy herd management.

INTRODUCTION

Dairy Herd Population Dynamics

Understanding the dynamics of the dairy herd population is crucial for decision-making and risk management in the dairy farming enterprise. Cows in a dairy herd follow probabilistic events of aging, culling, mortality, pregnancy, abortion, and calving.

The structure of a dairy herd at a given point in time (a snapshot) is a reliable indicator of the economic performance of the herd. Each cow in the herd at a particular time belongs to a specific category or "state" (e.g., second lactation at peak of milk or the dry period before calving for the third lactation) and each category or state has an estimated economic net return that can be calculated as the difference between the revenues and the expenses. For instance, the cow in peak lactation would have a greater net return than a cow during the dry period before calving.

The cow in peak lactation is producing a high amount of milk at the top of feed efficiency conversion while the dry cow is not producing any milk and still consuming feed for maintenance. The aggregation of the expected net returns of all cows in a herd makes up the herd economic performance at a given time.

Also, at a later time those cows in peak production might be or may become pregnant and reach a dry period whereas those dry cows might calve, start a new lactation, and reach a peak of production. The dairy herd structure changes every day and with it the herd economic performance. The challenge then is to find a farm-specific herd structure.

Markov-Chain Simulation

Knowing the probabilistic "transition" matrices that define culling, mortality, pregnancy, abortion and milk production, a herd can be simulated through Markov-chains (Cabrera et al., 2006) until the herd reaches a "steady state." Steady state is characterized by a constant herd structure that does not change over time.

Under the assumption that a farmer keeps the herd size constant (replacing cows leaving the herd) to make efficient use of the facilities, the herd will reach a steady state based on the transition matrices.

The steady state of the herd population will then become the "snapshot" to assess the economic performance of the herd. The exact same concept of the herd population steady state can be applied to understand a single cow's economic lifetime performance. The life of a cow can be described by a series of probabilistic events such as becoming pregnant, being culled or die (and replaced), have an abortion, reach a following lactation, and be in a particular production level. It is possible then to "follow" a cow's (and the replacement's) probabilistic life and to assess the lifetime economic performance of such a cow by aggregating all the probabilities of the cow being in a certain category or state and the net return the cow would produce in such a category. An exemplification of this processes can be found in Figure 1 that graphically shows the most important cow movements in a given herd structure. When the concept is to follow a cow through her lifetime it is reasonable to use a discount rate and make a net present value assessment: the economic net return later in the cow's life will have a lower value than the net return earlier in the cow's life. However, if the concept is to assess the herd economic net return, a net present value would not be needed because the analysis is performed at a given time and not through time.

This chapter is about herd population analysis as the main driver for decision-making in dairy farming. Whether it is a herd snapshot or a cow life time, the number of probabilistic categories or states in the conceptual model could become very large. Roughly, a simple monthly model for 10 lactations considering up to 24 months after calving and 9 months for pregnancy would have at least 2,400 possible states. The number increases quadratically for each additional defined state when the model is weekly or daily.

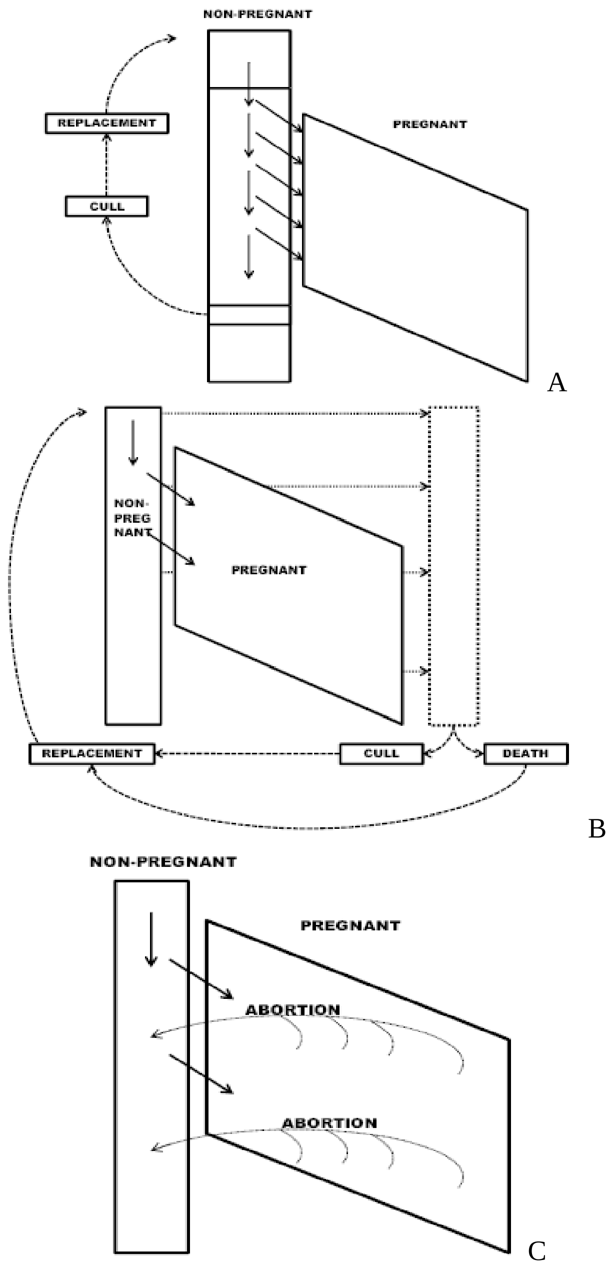

Figure 1. Graphic representation of a Markov-chain structure to be used in a dynamic program solved by linear programming. A: breeding process; B: The culling and mortality process; C: The abortion process.

Dynamic Transition Matrices

So far, we have discussed a simulation framework using Markov-chains solved to reach a steady state under the presumption that the probabilistic transition matrices are constant. It is important to recognize that the transition matrices could vary because of management, environmental factors, or any other uncertain factors. For analysis purposes, it is customary to assume that the involuntary culling rate, mortality rate, and abortion rate remain constant in the long term. However, three important transition matrices: 1) the probability of a cow to become pregnant (reproduction), 2) the farmer's decision to replace or maintain a cow for any other reason (voluntary culling and replacement), and 3) milk production, could certainly change more dynamically.

The Need for Dynamic Programming

The reproduction and replacement problems can be studied by using Markov-chains (St-Pierre and Jones, 2001). The framework has been to assess the whole herd net return under different reproductive or replacement scenarios. However, a more advanced framework requires an additional step that would optimize the decisions of replacement or reproduction. The basic questions to respond are: 1) Should the farmer keep this cow in the herd or would the net return of the herd benefit by replacing this cow? and 2) If the previous decision was to keep this cow and this cow is not pregnant, should the farmer continue the reproductive services on this cow?

The answers to the above questions are not trivial. Indeed, the answers require a detailed and systematic analysis. Although it has been around for long time, the Dynamic Programming (DP) (Bellman, 1957) framework is still the state-of-the-art technique to respond those questions. Dynamic programming is a sequential optimization in a Markov-chain structure (De Vries, 2004, 2006).

The Need for Linear Programming

Dynamic programming commonly has been solved by code writing (De Lorenzo et al, 1993). However, the DP problem can also be solved by using LP algorithms (Cabrera, 2010). Although the formulation of LP to solve DP has been available since the 1980s (Hillier and Lieberman 1986), this technique has not been used for practical applications. A first formulation of the dairy replacement problem was published in 1998 (Yates and Rehman, 1998); however that study included a model that was not a practical and realistic scenario for dairy farming. Cabrera (2010) is the first study with a DP/LP model for practical application.

The use of LP for solving a DP has many advantages over the other methods of solution. First, because of the complexity of the problem to be solved, the use of standard LP algorithms assures consistency and robustness in the solution. Second, the use of LP algorithms allows for sub-optimal solutions. Third, an LP formulation of a DP problem can efficiently manage different time spans in the Markov-chain dimensions of the model. Fourth, an LP formulation supports the inclusion of the interaction of herd mates.

Overview of the Next Sections

Following is a proposed general formulation of an LP problem to solve a DP optimization problem for dairy herd management in a matrix framework. Next is the discussion of two practical applications studied under the proposed framework.

FORMULATION OF LINEAR PROGRAMMING TO SOLVE A DYNAMIC PROGRAMMING OPTIMIZATION PROBLEM FOR DAIRY HERD MANAGEMENT

Mathematical Formulation

This is a general mathematical formulation of an LP/DP problem adapted from Cabrera (2010). The objective function is to maximize the net return of the decisions made, therefore:

$$\text{Optimum economic solution} = \max \sum_{i=1}^{I} \sum_{k=1}^{K} y_{ik} NR_{ik} \qquad [1]$$

where i is the category or state and k is the decision to be made, I is the total number of decision variables and K is the number total of possible decisions.

Then, y_{ik} is the steady state proportion of state i when decision k is made and NR_{ik} is the net return expected for the state i when decision k is made.

The constraints of the model are:

The non-negativity of all decision variables:

$$y_{ik} \geq 0 \text{ for all } i \text{ and } k \qquad [2]$$

the constraint that assures that the herd size remains constant:

$$\sum_{i=1}^{I}\sum_{k=1}^{K} y_{ik} = 1 \qquad [3]$$

and the constraints that assure the flow of cows through the possible categories or states to reach steady state of the herd population:

$$\sum_{k=1}^{K} y_{ik} - \sum_{i=1}^{I}\sum_{k=1}^{K} y_{ik} P_{ijk} = 0 \text{ for all } j \qquad [4]$$

where j is the number of rows (vertical axis) in the dimensions of the matrix and P_{ijk} represent the ij^{th} element of vector of transition probabilities resulting from making decision k. Therefore, the movement of cows is accounted for from one state to a successive potential state determined by the law of probabilities contained in the transition matrices of probabilities of involuntary culling, mortality, pregnancy, and abortion.

Matrix Formulation

As important as it is to understand the mathematical equations, it is also important to understand how a problem of this magnitude can be implemented for practical applications in a spreadsheet system. Implementation of LP models using spreadsheets is becoming more and more popular because 1) these can be better understood visually on spreadsheets, 2) these can be set up with relative ease, and 3) these can better accommodate the creation of decision support systems. Some limitations of using spreadsheets include the space available for dimensioning the model and the efficiency of the algorithms for solving the problem. Nonetheless, spreadsheet software is quickly improving capabilities to handle large models with lower computational requirements. In addition, the methodology and framework presented here is applicable for any software and solver algorithm. To facilitate understanding this section, a running example is introduced. This example is an over-simplification of a real situation, but allows one to understand and follow the important concepts needed for this framework. The Solver add in for Excel® is used in this example to solve the problem. For larger spreadsheet problems, a Premium Solver may be required.

Table 1 represents the running DP/LP matrix in a spreadsheet format. The objective function is to maximize net return (Cells Z5:Y5; Equation 1). The value of cells Z5:Y5 is the sum product of two vectors: the expected net return (cells A8:T8) and the decision cells (cells A5:T5). The constants of the model are represented in cells A10:T19. The optimization is performed under the constraints of non-negativity of the decision variables (Equation 2 inserted directly in the solver engine), the constraint of a constant herd size (Equation 3; cell Z9 = cell Y9), and the steady state herd population constraint (Equation 4; cells Z10:Z19 = Y10:Y19). The value of cell Z9 is the sum of the values of the decision cells (A5:T5) and the value of cell Y9 is a constant = 1. The value of each cell from Z10 through Z19 is the sum product of two vectors: the decision cells (cells A5:T5) and the constants (A through T) from the same cell Z. The value of each cell from Y10 through Y19 is a constant = 0.

1. Set The Dimensions of the Model

This is a critical step that will drive the rest of the work. There is a trade-off between complexity and precision. The larger the model, the larger the complexity and the more the computational resources required to solve it. On the other hand, an over-simplified model will not well represent practical dairy farm conditions. The dimensions of the model will determine the number of decision variables included in the LP model. The concept of dimensioning the model is better understood with the example. Let's use a simple model that would only include lactations as the state variables and would not sub divide lactations into smaller states. This simple model could consider 10 lactations as the maximum life time of a cow. Also, this example model considers that in every lactation after the first lactation, there is a decision of whether to keep or replace a cow.

Table 1. Running example of an equidistant linear programming matrix to solve a dynamic programming model for dairy herd decision-making

	Z	Y			A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
1					Lactation Number																			
2					Keep the Cow										Replace the Cow									
3					1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10
4	Max Net Return																							
5	$1,305				33.85%	25.39%	17.26%	11.05%	6.63%	3.78%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.04%	0.00%	0.00%	0.00%
6																								
7					Expected Net Return																			
8	Constraints				$1,000	$1,500	$2,000	$1,500	$1,000	$500	$300	$200	$100	-$400	-$500	-$500	-$500	-$500	-$500	-$500	-$500	-$500	-$500	-$500
9	1	1																						
10	0	0	Lactation Number	1	75%	-32%	-36%	-40%	-43%	-46%	-50%	-54%	-57%	-100%	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
11	0	0		2	-75%	1										1								
12	0	0		3		-68%	1										1							
13	0	0		4			-64%	1										1						
14	0	0		5				-60%	1										1					
15	0	0		6					-57%	1										1				
16	0	0		7						-54%	1										1			
17	0	0		8							-50%	1										1		
18	0	0		9								-46%	1										1	
19	0	0		10									-43%	-100%										1

Table 2. Running example of an event-driven linear programming matrix to solve a dynamic programming model for dairy herd decision-making

	Z	Y				A	B	C	D	E	F	G	H	I	J	K	L	M	N
1																			
2						Lactation Number													
3						Keep the Cow							Replace the Cow						
4						1	2	3	4	5-6	7-8	9-10	1	2	3	4	5-6	7-8	9-10
5	Max Net Return					1	1	1	1	0.5	0.5	0.5	1	1	1	1	0.5	0.5	0.5
6	$1,005					30.83%	23.12%	15.72%	8.49%	5.09%	0.00%	17.89%	0.00%	0.00%	0.00%	0.00%	0.00%	2.83%	17.89%
7																			
8						Expected Net Return													
9	Constraints					$1,000	$1,500	$2,000	$1,500	$1,500	$500	-$300	-$500	-$500	-$500	-$500	-$1,000	-$1,000	-$1,000
10	1	1																	
11	0	0		Lactation Number	1	0.9	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-1	-1	-1	-1	-1	-1	-1
12	0	0			2	-75%	1							1					
13	0	0			3		-68%	1							1				
14	0	0			4			-54%	1							1			
15	0	0			5-6				-60%	1							1		
16	0	0			7-8					-56%	1							1	
17	0	0			9-10						-45%	-100%							1

The dimensions of this model are: 10 lactations x 10 lactations = 100 states for deciding to keep the cow and 10 lactations x 10 lactations = 100 states for deciding to replace the cow. Therefore, the matrix would have 10 vertical and 20 horizontal cells with 200 cells in total that represent the constants or transition matrices that are the culling rates per lactation (Table 1: Columns A to T and Rows 10 to 19). The expected net returns and the decision cells are two additional rows in the matrix that also have 20 cells (Table 1: Columns A to T, rows 5 and 8, respectively). The constraints are two additional columns that have 11 cells: one for each lactation (Table 1: Z10 to Y19) and one to keep the herd population constant (Table 1: Z9 to Y9).

Generalizing, the matrix would have the number of defined states on the vertical axis and the number of defined states times the number of decisions on the horizontal axis:

Vertical axis = Number of defined states
Horizontal Axis = Number of defined states * number of decisions

The dimension of the model is then the vertical axis multiplied by the horizontal axis:

Dimension of the model (cells) = Vertical axis * horizontal axis or:
Dimension of the model (cells) = Number of decisions * (number of defined states)^2

Let's now estimate the dimensions of a monthly model that divides 10 lactations of up to 24 months after calving and the pregnancy up to 10 months (0 for non-pregnant cows and 1 to 9 for pregnant ones). The number of cells for one lactation would then be 24 months * 10 pregnancy states = 240 and the number of cells for 10 lactations would then be 2,400. Considering only 2 decisions, keep or replace, the number of cells of this monthly model would be 2 * 2,400^2 = 11,520,000 constants with a vertical axis of 2,400 constraints and a horizontal axis of 4,800 decision variables. The estimated dimensions of a weekly model that divide 10 lactations in up to 108 weeks and the pregnancy in up to 40 weeks with 2 decisions would be: 2 * 43,200^2 = 3,732,480,000 with a vertical axis of 43,200 cells and a horizontal axis of 86,400. In similar fashion, the dimensions of a daily model that divides 10 lactations in up to 720 days and the pregnancy in up to 280 days with 2 decisions would be: 2 * 2,016,000^2= 8,128,512,000,000 with a vertical axis of 2,016,000 cells and a horizontal axis of 4,032,000.

The dimensions of the model grow quadratically and rapidly become too large to be manageable and solved (as a reference, the standard solver engine shipped with spreadsheets can handle only 200 decision variables (horizontal axis)). Larger capacities are, of course, available. The researcher needs to make a judgment call to decide on the dimensions of the model. It was assumed above that for every state there exists a decision, but that might not always be the case. It is unlikely, for example, that a cow would be "voluntarily" replaced when pregnant or in very early lactation. Therefore, there might not be a need to include a decision for replacement when a cow is pregnant or in early lactation.

Under the same rationale, if the decision would be whether to continue the breeding program or not, this should only apply to not pregnant cows and a certain defined time after calving as is the standard practice. Therefore, there are ways to "save" some space in the dimensions of the model. Nonetheless, the difference in dimensions between monthly, weekly, or daily models is still large.

So far, the assumption has been that the model needs to be equidistant in the time spans. However, one of the advantages of using LP to solve DP is the possibility to define models with different time steps: e.g., one step could be a month and the next one could be a week. Event-driven LP models can eventually accommodate any combination of time steps with savings in model dimensioning and consequently, computational efficiency.

The challenges in event-driven models are the internal transfers between states that have different dimensions. If a state needs to transfer to several states with different dimensions, it would require additional dimensions, which decreases the advantage of using event-driven models. For an example, see Table 2. This is a follow up of the running example introduced before.

Notice that lactations 1 to 4 remain as before, but the rest of lactations have been aggregated into sets of 2 (5 and 6, 7 and 8, and 9 and 10). Therefore, instead of 10 states, there are only 8 states per decision and the dimension of the model is now 2 x 8 x 8 = 128 or 62 fewer cells than originally.

The results with this model are comparable, though not the same, as the original. These differences are due to the need for averaging the transition matrices and the large steps. In practical applications, smaller differences would still be expected, but with the same trend of results when performing sensitivity or scenario analyses.

2. Define the Transition Matrices

Several pieces of information are needed to properly analyze dairy herd systems. For each defined variable, a transition matrix with the dimensions of the model is needed. Following are a list, definition, and examples of the most important transition matrices needed.

2.1. Involuntary Culling and Mortality

A cow always has a probability of being involuntarily culled from the herd. Involuntary culling is when a cow leaves the herd because of unforeseen reasons out of the control of the herd manager. This could happen, for example, when a cow is badly injured or sick. This is different than voluntary culling when the herd manager decides to cull a cow because of low production, reproductive failure or any other reason. Voluntary culling can be optimized by using the model framework described here, which could find if it is better to replace or to keep a cow. However, involuntary culling is an inherent characteristic of the herd. Involuntary culling can be assumed to be relatively constant and can be collected from herd historical records or from published reports. The transition matrix of involuntary culling for the running example can be seen in Table 3. In order to maintain the herd population constant, culled animals are replaced the next period in the dimensions of the matrix.

Some proportion of cows might die on the farm and are also leaving the herd involuntarily, but differently than involuntarily culled animals. Dead animals do not have any return and may even incur a disposal expense (which is discussed in the next section). The transition matrix of mortality rate of the running example can be seen in Table 3.

For effects of herd population, there is no distinction between involuntary culling and mortality. Both can be aggregated to estimate the proportion of cows leaving the herd in a particular state. Take a look at the example matrix (Table 1) and the involuntary culling and mortality rate matrices (Table 3): The proportion of cows leaving the herd in lactation 4 is 40% (denoted by -40% in cell D10), therefore a proportion of 60% of the cows will move to the fifth lactation (denoted by -60% in cell D14). This is set up for all states in the matrix. Note that the first and the last lactations have a different set up. For the first state, the involuntary culling and mortality are directly applied without a transfer, and are the number ones in the matrix to save dimensioning space. Note that for the last lactation 100% of replacement is applied. This is discussed later in the set up of the model.

Table 3. Involuntary culling, mortality rate, and total cows leaving the herd per lactation used for the running example

		Involuntary Culling (%/lactation)	Mortality Rate (%/lactation)	Cows Leaving the Herd (%/lactation)	Cows Remaining in the Herd (%/lactation)
Lactation Number	1	20%	5%	25%	75%
	2	25%	7%	32%	68%
	3	28%	8%	36%	64%
	4	31%	9%	40%	60%
	5	33%	10%	43%	57%
	6	35%	11%	46%	54%
	7	38%	12%	50%	50%
	8	40%	14%	54%	46%
	9	42%	15%	57%	43%
	10	45%	20%	65%	35%

2.2. Reproduction and Abortion

Reproduction parameters determine the probability of a cow becoming pregnant and when pregnant, the probability of abortion. These dimensions are not considered in the running example because the state "lactation" was not further subdivided. However, we can use Figure 1 to describe the process of incorporating reproduction and abortion parameters.

A cow starts the lactation as a non-pregnant cow, after a certain defined time, a reproductive program is applied to the cow (e.g., first reproductive service at 70 days after calving). Therefore, a proportion of those cows not pregnant and not leaving the herd become pregnant. The cow population is divided into non-pregnant and pregnant cows and each group has different characteristics regarding culling and expected net returns. Pregnant cows have an estimated time for calving and move to the next lactation if not aborting, being culled, or dying.

Pregnant cows could abort and return to the stream of non-pregnant cows. Non-pregnant cows will have successive reproductive attempts until either they get pregnant or they are culled for reproductive failure. Similar transition matrices as those for the culling and mortality are needed to define the probability of pregnancy and abortion according to the dimensions of the model. This is discussed more in detail with the applications.

2.3. Milk Production

Milk production is the most important economic variable for dairy herd decision making. The lactation curves could be defined as a table following the dimensions of the model or they could be defined as a function of the days after calving depending on the herd milk production rolling herd average. Along with lactation curves, feed consumption is also. Normally, there is an interaction of pregnancy and milk production.

3. Define the Expected Net Returns

Every cell in the horizontal axis of the matrix must have an expected net return (ENR) as shown in the example matrix (Cells A8 to T8). Five factors are critical in this calculation: milk income over feed cost (IOFC), involuntary culling cost (ICC), mortality cost (MC), reproduction cost (RC), and income from calving (new born) (IC).

$$ENR = IOFC - ICC - MC - RC + IC \qquad [5]$$

3.1. Income Over Feed Cost (IOFC)

As its name indicates, this is the difference between milk income and feed cost. Milk income is the product of milk price (MP) and milk production (MQ). The milk price is defined by the analyst according to farm and market conditions. Milk production is the accumulated milk produced during a defined time step in the dimensions of the matrix. The feed cost is the feed price (FP) multiplied by the feed amount (FQ).

$$IOFC = MP * MQ - FP * FQ \qquad [6]$$

3.2. Involuntary Culling Cost

When a cow is culled the farmer recovers a salvage value (SV, meat value of the cow), but incurs the expense of buying a replacement (RC) that usually is a pregnant heifer ready to calve. Therefore the cost of bringing the heifer to the herd will be partially offset because of the value of the new born (IC).

$$ICC = SV - RC + IC \qquad [7]$$

3.3. Mortality Cost

When a cow dies, the herd incurs a cost of disposal (CD) and the cost of buying a replacement (RC). As above, these costs are partially offset by the value of the new born (IC) coming with the replacement.

$$MC = - CD - RC + IC \quad [8]$$

3.4. Reproduction Cost

Reproduction costs can include labor (L), semen dose (SD), hormones (H), and pregnancy diagnosis (PD). These costs can be calculated based on herd records:

$$RC = - L - SD - H - PD \quad [9]$$

3.5. Income From Calving

The value of a calf can be calculated as the sum product of the proportion of male (ML) and female (FL) offspring and their respective values (MLP and FLP)

$$IC = ML * MLP + FL * FLP \quad [10]$$

4. Setting Up the Model

The model can be set up in different ways. One way is the running example (Tables 1 and 2). More important is to keep in mind the concept of the functioning of the model. The herd population follows Markov-chain probabilities defined by the matrix and the culling and the mortality transition matrices. For simplicity, no reproduction events are simulated: The assumption is that every cow has an average reproductive performance for a lactation. Also, for each lactation, the expected net return (cells A8 to J8 in Table 1) is already calculated. The cost of replacing a cow (cells K8 to T8 in Table 1) is also calculated for each column in the matrix. In addition, the involuntary culling and mortality rates (from Table 3) are represented in the part of the matrix pertaining to involuntarily replacing or keeping the cow (cells A10 to J10 and cells A11, B12, C13, D14, E15, F16, G17, H18, and I19 to J19 in Table 1).

It is important to understand how the net returns are set for the decision of keeping the cow. The data indicate that 25% of first parity cows leave the herd. Consequently cell A10 in the matrix indicates that 75% of first parity cows end the first lactation. Cell A11 indicates that this proportion of cows is then transferred to second lactation (-75%). The negative sign, as in a balancing accounting book, indicates the proportion of cows that will be removed from first lactation and added to the second lactation. The transfer is achieved differently in following lactations. In lactation 2 for example, the data indicate that 32% of second parity cows leave the herd and consequently 68% of cows finalize second lactation and reach third lactation. Cell B11 (the number 1) is a "receptor" of the cows coming from first parity and at the same time is a "distributor" of cows either leaving the herd (-32%, cell B10) or cows moving to the third lactation (-68%, cell B12). Note that 32% of the cows leaving the herd in the second lactation are replaced to the first parity (row 10). One extra row could have been used and one extra column to do a similar transfer to first parity, but those cows leaving the herd during the first lactation would return as replacement to the first lactation, which can be accounted more efficiently by entering 75% in cell A10 and -75% in cell A11 (25% of cows leaving the herd in first lactation). The same procedure of second lactation transfer is applied to all successive lactations except the last one (cells J10 and J19). The assumption is that all cows reaching lactation 10 are replaced. Once again, there could be an additional transfer, but in order to save dimensioning space in the running model, this replacement is forced. Results are not changed because with the transition probabilities and the dimension of the model, cows will not reach that state.

Now, let's discuss the decision of voluntary replacement (right side block of the matrix). First, for each replacement, a cost is calculated that will be similar to the involuntary culling cost (ICC). If the replacement is voluntary, the farmer would incur the cost of buying a replacement cow that will be a pregnant heifer, so the cost of the replacement animal is partially offset by the new born (coming with the heifer) and the salvage value received for the culled animal. This is calculated to be $500. Consequently, that value (negative) is used in cells K8 to T8. Now, in the matrix, it is necessary to connect the transfers that will let the model perform a decision at each step of the solution. For instance, let's focus on cells with the number 1 (positive 1) between L11 and T19, which correspond to the lactation rows (2 to 9). Also note cells L10 to T10 that have a -1 (negative 1). At the transfer moment between first and second lactation, the model has two options, the first option has been discussed above: They continue their normal course to third lactation (B11 transfer center). However, there is another option, to move the cow out of the herd (L11 transfer center). In column B, the net return is $1,500 while in column L the net return is -$500. The rationale applies equally for successive lactations. The LP algorithm finds what has a higher net return in the long-term, whether to keep the cow one more lactation or replace a cow at the end of this lactation. As the model is set up in the running example, columns K and T could have been avoided. The decision to replace a cow starts in second lactation (column K) and column J is already transferring all cows finalizing lactation 10. Those columns were left in the matrix only for demonstration purposes. As previously discussed, they do not affect the functioning or results of the LP model.

The next step is to enter the equations in the spreadsheet. These equations will work with the Solver iterations. First, it is necessary to define where the results are going to be displayed. There will be as many result cells (decision variables) as the horizontal dimension of the matrix. For convenience, cells A5 to T5 are selected to be the result cells.

A condition is to calculate the net return that will be the maximization target of the LP problem (Equation 1). This is displayed in merged cells Z5 and Y5. Merged cells Z5 and Y5 have this equation: =SUMPRODUCT(A8:T8,A5:T5). The function "SUMPRODUCT" in a spreadsheet performs the sum of the multiplication of two arrays. In this case this is the sum of the multiplication of each lactation net return by the proportion of cows in each lactation (the variables resulting from the solution) after reaching a steady state.

Another condition required is that all decision cells (results) be non negative (Equation 2). This is set up directly in the solver engine of a spreadsheet either as an equation (cells A5 to T5 ≥ 0) or as an option provided by the solver engine.

Another condition of the problem to be solved is that the herd population must remain constant across time (Equation 3). This condition is set by entering one equation and a constant in the spreadsheet. Cell Z9 contains this equation: =SUM (A5:T5) and cell Y9 is equal to the number 1 (a constant). Later, these two cells are used when setting up the constraints of the LP model in the solver engine.

Another important condition is that the Markov-chain model defined in the LP will reach a steady state (Equation 4). This condition is set by a number of equations. As many equations as the vertical dimensions of the LP model. These are displayed in cells Z8 to Z19. Cell Z8 has this equation: =SUMPRODUCT(A10:T10,$A$5:$T$5). The two arrays in this particular equation are the first lactation (A10 to T10) and the variables of solution (A5 to T5): A10 * A5 + B10 * B5 + ...+T10 *T5. Note that for the second array the dollar ($) signs are used in order to fix this array in the equation when copied to cells below. In each cell from Z9 to Z19 the equation has as a first array the cells A to T with its corresponding row number and as second array the cells A5 to T5.

A final step in the set up is to define the LP problem, which requires the definition of three conditions: 1) the target variable, 2) the changing variables (decision variables), and 3) the constraints. Different spreadsheet software would have different ways to set up the LP problem. Therefore, a general way to set up the example model is provided, which would apply to any LP algorithm and any spreadsheet software.

1) Target variable: maximize cell Z5
2) Changing variables (decision variables): cells A5:T5
3) Contraints: a) cell Z9 = Y9 and b) cells Z10:Z19 = Y10:Y19. Note that Y10 to Y19 have no equation in them; they are zeroes, but they could also just be blank cells.

5. Solve the Model

The model is ready to be solved. After the LP problem is defined in the solver engine, it is usually solved by an option named "solve." Once the solve option is applied, the solver engine starts a number of iterations until finding the maximum target variable value by changing the indicated cells, under the constraints imposed.

For the running example, this means that the solver engine will look for the maximum net return when reaching a steady state of the herd population by finding the optimal replacement time. The results will include the maximum net return (cell Z5) and the proportion of herd population in different lactations (cells A5 to T5).

6. Analyze the results

The solution to the example problem is seen in Table 1. This indicates that the maximum net return that could be obtained is $1,305/cow when the replacement policy indicates to keep cows only until lactation number 6. The DP problem solved through LP found that a cow should be replaced at the end of the sixth lactation. By doing so, the herd population is distributed with

33.85% in first lactation, 25.39% in second lactation, 17.26% in third lactation, 11.05% in fourth lactation, 6.63% in fifth lactation, and 3.78% in sixth lactation. The remainder of the herd population, 2.04% (cell Q5), is being replaced when completing lactation 6. No other configuration of the herd will give a higher net return.

APPLICATIONS USING LINEAR PROGRAMMING FOR DAIRY HERD SIMULATION AND OPTIMIZATION

1. Monthly Model

Cabrera (2010) is a comprehensive model that was used to study the interaction of herd economics, feeding diets, and nitrogen excretion. This model was set monthly for 15 lactations having 10 states for pregnancy (0 for non-pregnant, and 1 to 9 pregnancy months). Figure 2 displays a diagram that represents the cow flows on the dimensions of the model. The dimensions of the model were 2,790 rows by 5,580 columns (15,568,200 cells) having 5,580 decision variables (one for each column). The LP algorithm was used to find the maximum net return for the decision of keeping or replacing a cow in each state under a specified feeding diet.

For each category or state included in the model there was a culling risk, a mortality risk, a pregnancy risk, milk production, and amount of nitrogen (N) excretion depending on the diet fed. Also, for each category or state, the model included a net return based on the five factors discussed earlier: IOFC, ICC, MC, RC, IC, and in addition an environmental factor resulting from the interaction of the value of the N excreted and the cost of spreading the manure on crop fields. The model was used to study five diet treatments (an all forage diet and a high concentrate diet with three other intermediate diets).

The model was solved for each diet and the results indicated the herd structure at steady state and the replacement policy to find the maximum net return with or without the constraint of a maximum allowable level of N excretion. The model was solved using the Risk Solver Platform (Frontline Solvers, Incline Village, NV, USA) and each solution took approximately 10 minutes with a 6.00 GB (RAM), 64-bit Operating System, and a computer with two 2.8 GHz processors.

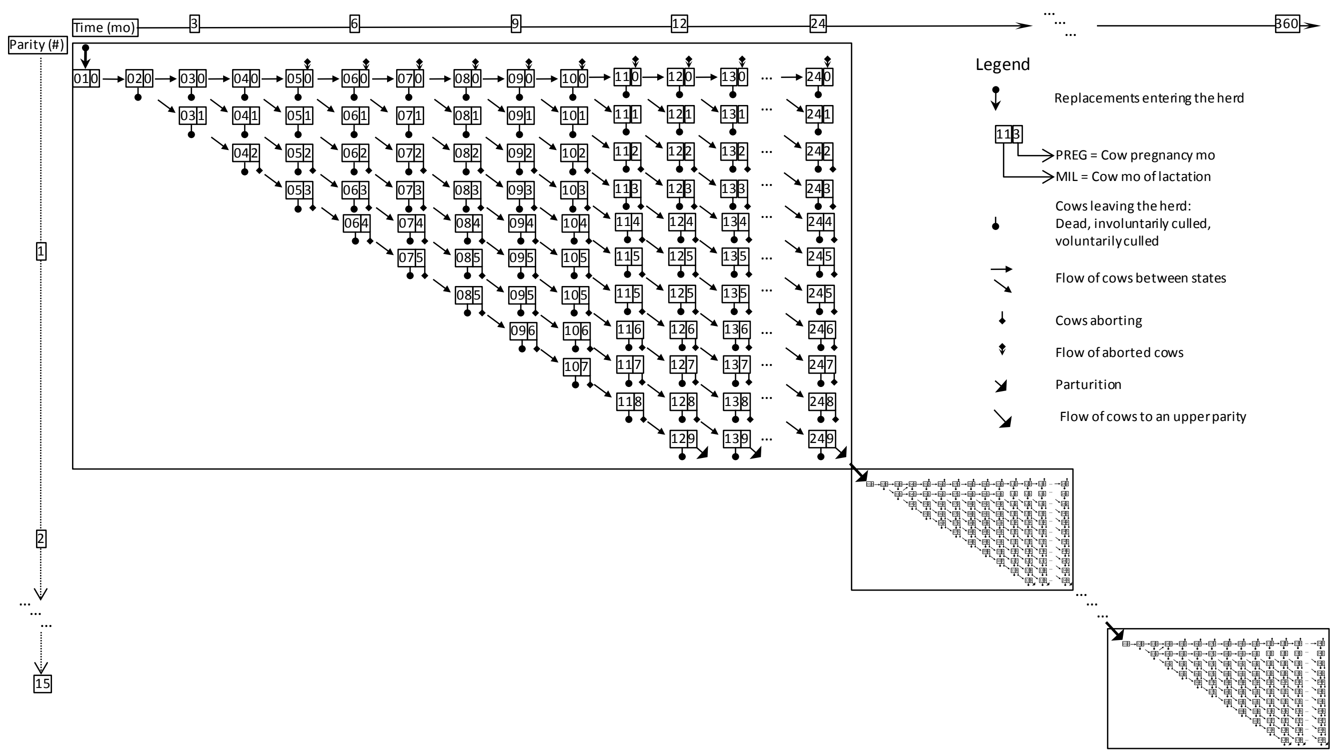

Figure 2. Graphic representation of Markov-chain probabilistic processes of cow flow transitions of a linear programming model used to solve the dynamic problem of feeding diet groups in monthly steps in Cabrera (2010).

The results indicated that the optimal policy would be to replace non-pregnant cows at a certain month in lactation depending on market conditions, lactation number, and diet. Pregnant cows should not be replaced.

With reasonable prices and market conditions, the replacement of non-pregnant cows should occur at 11 months after calving for first lactation cows and at 10 months after calving for later lactation cows. Replacement could be performed one month later if the diet was all-forage. Imposing a maximum N excretion of 12 kg of N excreted per cow per month drastically changed the dynamics of the replacement policy: Cows consuming concentrates would be replaced 2 months earlier.

The resulting herd structure indicated that on average around 50% of the cows would be in first parity, 24% in second parity, 12% in third parity, 6% in fourth parity, and only 8% in fifth to fifteenth parity. Thus, the herd structure was dependent on the optimal policy found by the model.

The author concluded that the implementation of a Markov-chain LP model to solve a DP problem is feasible for practical decision making in dairy farming and that this was an important advancement for dairy decision-making that provides both robustness and versatility in operations research. The model could become a valuable tool to support economic decision-making of dairy herd management.

2. Event-Driven Model

In order to gain efficiency in the solution and reduce the dimensions of the model, event-driven models can be set up. One example is an adaptation of the monthly model of Cabrera (2010) to events dictated by the reproductive events. Cows becoming pregnant, if not aborting, involuntarily culled, or dying, will reach a next parity after the gestation process. Therefore, substantial savings in the dimensions of the model can be obtained by moving pregnant cows from one parity to the next in one step. Cows not becoming pregnant will be bred again in a certain period of time (next estrous cycle or next timed or programmed breeding).

The model then can use the LP algorithms to decide if a cow should be bred again or replaced at a certain time. This model was solved using the Risk Solver Platform (Frontline Solvers, Incline Village, NV, USA) and each solution took approximately 30 seconds with a 6.00 GB (RAM), 64-bit Operating System, and a computer with two 2.8 GHz processors.

The savings in the dimensions are substantial, but some challenges remain.

First, reproductive programs are highly variable, so the model needs to be dynamically set up depending on the reproductive program parameters (e.g., the model is different if the inter-breeding interval is 42 or 35 days).

Second, together with the dynamic steps of the model, the transition variables need to accommodate these reproductive parameters (e.g., the milk production or culling risk need to be accumulated dynamically to the dimensions of the model).

Third, the internal transfers between categories or states could be difficult to handle. This is especially important for aborting cows. Cows aborting are moved back to the stream of non-pregnant cows, which are running at different time steps. If the transfer would occur only once during a pregnancy, this could easily be managed by accounting for the time difference between the two flow streams. However, abortion rates change across gestation and then several events are needed to transfer them to the non-pregnant categories or states.

As previously discussed, for an exact account of them, there would be a need of additional columns and rows in the matrix, which deters the benefit of using an event-driven framework. One option is to accumulate abortion rates and return them at the available times in the stream of non-pregnant. This is an approximation that probably would have only minimal consequences on the results and this is the approach used for Cabrera (unpublished) in a reproductive event driven model.

Cabrera (unpublished) uses 12 lactations of reproductive events. The first event is the time to the first breeding. If the cow becomes pregnant, the next event is gestation. After gestation, calving, and after calving the next lactation if the cow did not abort, was culled, or died. If the cow did not get pregnant, the next event is the next breeding. The cow theoretically has the opportunity to have up to 24 breeding services and the model selects the number of optimal services in each lactation to maximize the net return. The model then has 540 rows by 792 columns (427,680) with 792 decision variables (one in each column). This model is 2.75% the size of the previous model (Cabrera, 2010) with substantial savings in solution efficiency. Results are expected to be approximately similar to the previous model, but with the main advantage that this model can study very detailed reproductive programs and their impact on the herd net return. Cabrera (2010) had the need to aggregate reproductive events monthly with loses in the sensitivity of the timing and efficiency of the reproductive programs.

CONCLUSION

Linear programming is a flexible tool that provides a response to difficult questions in the area of dairy herd management. In particular, linear programming is a feasible and efficient framework to solve a dynamic programming problem, which is the state-of-the-art in dairy herd economic decision-making. As demonstrated in this chapter, linear programming can be used for practical applications. The use of linear programming for dynamic programming has a series of advantages that include the solution for sub-optimal conditions, event-driven models, and the inclusion of the interaction of the performance of other cows in the herd (not demonstrated in this chapter). This chapter helps the reader to set up a linear programming matrix to solve a dynamic programming model using common spreadsheet software and standard solver algorithms. Although the running example presented here is an over-simplification of real herd conditions, the same concepts and framework are valid to set up models for practical applications for herd decision making. This is demonstrated with recently developed studies. Although the examples referred to the most relevant decisions in dairy herd population dynamics (replacement and reproduction) the framework can be applied to any other optimization scheme.

Finally, although this chapter proposes one way to set up the matrix, this can be set up multiple ways according to user preferences. However, regardless of the matrix set up, the results should be the same because for a given set of variables and constraints a linear program model should have one and only one solution.

Finally, the researcher will also need to decide whether to use an equidistant or an event-driven model. For a feed group study, a monthly model as discussed in the applications would be appropriate, but for a reproductive study, an event-driven model, could be preferred.

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