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A large Markovian linear program to optimize replacement policies and dairy herd net income for diets and nitrogen excretion

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ABSTRACT

The purpose of the study was 2-fold: 1) to propose a novel modeling framework using Markovian linear programming to optimize dairy farmer–defined goals under different decision schemes and 2) to illustrate the model with a practical application testing diets for entire lactations. A dairy herd population was represented by cow state variables defined by parity (1 to 15), month in lactation (1 to 24), and pregnancy status (0 nonpregnant and 1 to 9 mo of pregnancy). A database of 326,000 lactations of Holsteins from AgSource Dairy Herd Improvement service (http://agsource.crinet.com/page249/DHI) was used to parameterize reproduction, mortality, and involuntary culling. The problem was set up as a Markovian linear program model containing 5,580 decision variables and 8,731 constraints. The model optimized the net revenue of the steady state dairy herd population having 2 options in each state: keeping or replacing an animal. Five diets were studied to assess economic, environmental, and herd structural outcomes. Diets varied in proportions of alfalfa silage (38 to 98% of dry matter), high-moisture ear corn (0 to 42% of dry matter), and soybean meal (0 to 18% of dry matter) within and between lactations, which determined dry matter intake, milk production, and N excretion. Diet ingredient compositions ranged from one of high concentrates to alfalfa silage only. Hence, the model identified the maximum net revenue that included the value of nutrient excretion and the cost of manure disposal associated with the optimal policy. Outcomes related to optimal solutions included the herd population structure, the replacement policy, and the amount of N excreted under each diet experiment. The problem was solved using the Excel Risk Solver Platform with the Standard LP/Quadratic Engine. Consistent replacement policies were to (1) keep pregnant cows, (2) keep primiparous cows longer than multiparous cows, and (3) decrease replacement rates when milk and feed prices are favorable. The optimal policy called for the replacement of open cows between 7 and 12 mo in lactation depending on parity, diet, and market conditions. Under favorable market conditions, net revenue was greatest with the greatest concentrate diet, which was $15.24 and $52.32/mo per cow greater than the optimal net revenue realized with the intermediate and the no-concentrate (all-forage) diets, respectively. A suboptimal solution to limit the N excretion to 12 kg/mo per cow when market conditions were favorable resulted in a diet with the second-greatest amount of concentrates being the one with the greatest net revenue. Under unfavorable market conditions, the diet with the greatest concentrate content had the least net revenue compared with all the others. A suboptimal solution for a maximum N excretion of 12 kg/mo per cow with unfavorable market conditions resulted in the least-concentrate diet having the greatest net revenue ($22/mo per cow), followed by the second-greatest concentrate diet ($20/mo per cow) and the all-forage diet ($18/mo per cow). The implementation of a Markovian linear program for dairy decision making provides both robustness and versatility in operations research. The model could become a valuable tool for economic decision making for dairy farms.

Key words: replacement policy, system analysis, diet optimization, dynamic programming

INTRODUCTION

Economic optimization of dairy herd performance is a topic with a series of research questions that still require substantial investigation. For instance, the voluntary cow replacement problem and the question of optimal number of reproduction services continue to be answered inadequately. Although voluntary culling decisions are critical, they are usually based on intuition and not on a systematic economic analysis because of the lack of metrics and methods that could certify that decisions are correct.

Since the 1980s, dynamic programming (**DP**) based on Bellman's principle of optimality (Bellman, 1957) has been the most recognized method to optimize dairy herd economics. Several studies have used DP for dairy economic decision making, advancing the knowledge of

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dairy herd optimization. For example, van Arendonk (1984, 1985, 1986, 1988) and van Arendonk and Dijkhuizen (1985) studied replacement and reproduction policy in dairy cattle. Kristensen (1987) introduced the concept of hierarchic Markovian processes and reformulated the concept of policy and value iteration with substantial gains in computational efficiency to solve DP problems (Kristensen, 1988, 1991). Later Kristensen and Thysen (1991) developed a model that accounted for milk quotas in Europe, and in the United States, DeLorenzo et al. (1993) developed the largest DP model at the time with 151,200 cow states to solve the replacement problem. In addition, Kristensen (1993) combined Bayesian probabilities with Markovian decision processes in a DP model to calculate optimal replacement rates, and Stott (1994) found by DP that keeping milking cows longer in the herd would increase the net profit of British dairy farms. More recently, De Vries (2004, 2006) has been applying DP and Markovian simulation processes together to solve and simulate the replacement and reproduction problems, offering interesting advancements in the decision process. De Vries (2004) provided a new algorithm to account for seasonally delayed replacement of cows, and De Vries (2006) provided new algorithms to calculate the value of a pregnancy.

However, a potential problem of traditional DP formulation is that the model can easily become very large and complicated with limited applicability to real-life problems (Smith et al., 1993). An additional challenge with the traditional DP formulation is the insufficient number of parameters selected by the decision maker, which limits its applicability (Groenendaal et al., 2004). Lehenbauer and Oltjen (1998) stated that more effort has been devoted to constructing models than to applying these models in real-life farm decision making. For these reasons, Groenendaal and colleagues (2004) found it worthwhile to revisit the marginal net revenue technique with the justification that this simpler method could easily be applied in final users' decision making through decision support systems without compromising the accuracy of the outcomes. An appealing method that simplifies the search for the optimal policy was described by Hillier and Lieberman (1986). The method is the solution of the original Bellman equation by using linear programming (**LP**).

A formulation of the DP problem as an LP algorithm would allow the inclusion of interaction between herd mates in a particular herd. Yates and Rehman (1998), in the only study of its class, used an LP formulation of the Markovian decision process for the dairy replacement problem accounting for the performance of the whole herd in addition to the performance of each individual animal. The Yates and Rehman (1998) model was rather small, having only parity as the state variable (1 to 12), and consequently, their results could be used as good conjectures but not as conclusive findings.

Linear programming formulation of DP problems overcomes another limitation of the traditional DP method. The LP formulation allows the possibility of solving a problem for user-defined suboptimal conditions. The suboptimal condition of a DP formulation has been referred to as an unsolvable limitation of traditional DP formulation (Dekkers, 1991) and has remained unsolved hitherto in the literature. Another advantage of LP formulation over traditional DP formulation is that once the problem has been defined, standard LP optimization algorithms can solve the problem efficiently to explore several different research questions. The formulation of a Markovian LP problem allows the investigator and hopefully the end users to better interact with the model for decision making. The solution of a DP problem as an LP problem would also allow efficient management for different time steps in the dynamic processes. The goal of this study was to formulate a large Markovian LP model capable of solving a real-farm dairy herd replacement DP problem including suboptimal solutions.

MATERIALS AND METHODS

Production Scenario

A typical adult Holstein herd was represented and optimized. In the model, a cow started its productive life after first calving. Each cow produced milk for a variable length of time depending on its reproductive status, probability of survival, and probability of involuntary culling. A pregnant cow (**PREG**) was dried off 2 mo before expected calving. At calving, the cow was assigned to the next parity (**PAR**) category and started a new productive cycle. When a cow had 2 mo in lactation (**MIL**), a reproductive program was started and continued through successive pregnancy service attempts. The pregnancy rate determined the probability of a cow becoming pregnant in a month. If conception failed, cows continued to be serviced in subsequent estrus cycles. This protocol repeated for each parity under this structure. The model selected the optimal time and cow state of voluntary culling that maximized net revenue and found the herd steady state. By implication, the model also decided the optimal number of reproductive services that should be performed.

The adult population of cows in a herd could be realistically described as successive steps through changes of states established by transition probabilities (Cabrera et al., 2006a, 2008). The time step in the model was 1

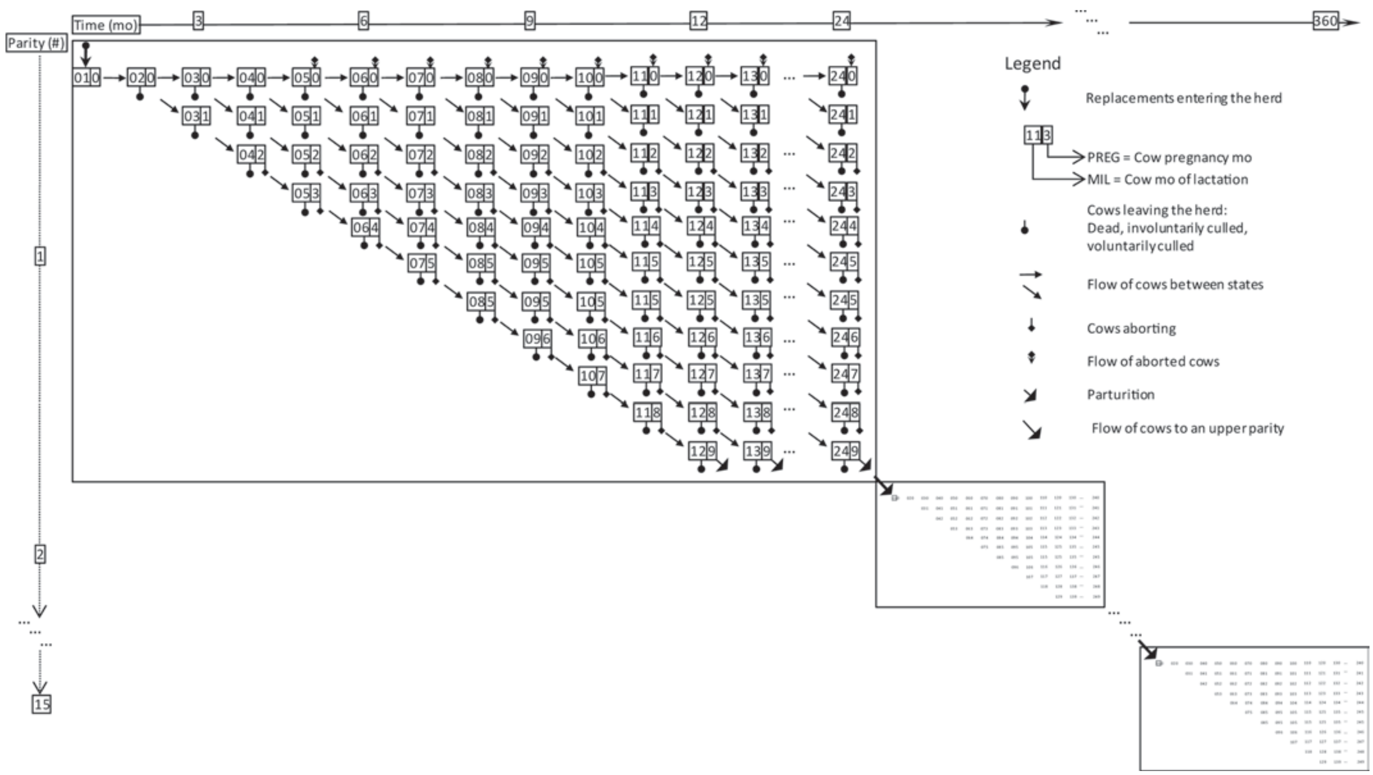

Figure 1. Representation of probabilistic Markovian processes of cow flow transitions.

mo; consequently, all production (e.g., milk), economic (e.g., milk price), and biological (e.g., pregnancy rate) parameters were defined on a monthly basis. Therefore, every month, every cow in the herd was run through a probabilistic process that included several potential options. For example, the following processes were defined by transition probabilities, which were repeated for parities 1 to 15 (Figure 1). In every state and stage, there was always a possibility for a cow to be involuntarily culled or to unexpectedly die. When a cow survived from one state to the next, the cow became one MIL greater (except when the cow calved and became one parity older; Figure 1). When a cow was eligible for breeding, the cow could become pregnant, and when the cow was pregnant, the cow could abort.

In addition to the transition probabilities and potential states exemplified in Figure 1, cows could be voluntarily culled in any state at any point in time. Voluntary culling was handled by the model as the decision variable to maximize the net revenue.

The dimensions of the model were large enough to accommodate all potential cow states. These could be represented by vector matrices: PAR = 1 to 15, MIL = 1 to 24, and PREG = 0 (nonpregnant) and 1 to 9 (monthly pregnancy states). Therefore, there were $15 \times 24 \times 10 = 3{,}600$ possible states. However, to comply with the reproductive program that starts in MIL = 2, the MIL has to be at least 2 units greater than the pregnancy status (MIL > pregnancy state + 2), 54 possible states were excluded from each parity and 810 states for the 15 parity combinations. Consequently, the effective number of cow states was 186 per parity and 2,790 for all 15 parities.

The model started with one cow in PAR = 1, MIL = 1, and PREG = 0. Then, the stochastic Markovian processes, using transition probabilities, distributed the proportions of such cow to all potential states in the model. Through recursive iterations, this distribution across states reached a steady state. This iterative process was solved by standard LP algorithms.

Markovian Decision Processes and LP

The population dynamics in a dairy herd were described as Markovian processes. Markovian processes are a special case of stochastic processes in which it is possible to analytically track the stochastic processes through the Markovian property that indicates the conditional probability of any future event given any past event (Hillier and Lieberman, 1986). A Markovian decision model is then used to find a decision policy to optimize net revenues.

The model was then solved by LP as a modified Markovian dynamic program problem described as follows. The objective function maximized the net revenue of the vector decision made in each state. The matrix had 5,580 terms (2 decisions in each state by 2,790 states). Consequently, the solution vector had 5,580 terms. Therefore,

$$\text{Optimum economic solution} = \max \sum_{i=1}^{2{,}790} \sum_{k=1}^{2} y_{ik} NR_{ik}, \quad [1]$$

where i is the state and k is the decision to be made (1 = keep and 2 = replace). Consequently, y_{ik} is the steady state proportion of state i when decision k is made, and NR_{ik} is the net revenue expected for the state i when decision k is made. For example, y could be 0.003 and NR could be \$63.46 for $i = 222$ and $k = 1$. This means that the proportion of cows in PAR = 2, MIL = 9, and PREG = 6 ($i = 222$) would be 0.003 when the herd reaches steady state. This proportion of cows (group of cows) would produce a monthly net revenue of \$63.46. Equation 1 found the maximum NR of the optimum replacement policy. The optimum replacement policy indicated the optimal time of replacing a cow to maximize the NR depending on the states of MIL, PREG, and PAR.

The model had 8,731 constraints: the constraints of nonnegativity of all decision variables,

$$y_{ik} \geq 0 \text{ for all } i \text{ and } k, \quad [2]$$

the constraint that ensured that herd size remained constant so the sum of proportions at steady state were equal to 1,

$$\sum_{i=1}^{2{,}790} \sum_{k=1}^{2} y_{ik} = 1, \quad [3]$$

and 2,790 constraints (one for each state) that found the steady state probabilities,

$$\sum_{k=1}^{2} y_{ik} - \sum_{i=1}^{2{,}790} \sum_{k=1}^{2} y_{ik} P_{ijk} = 0 \text{ for } j = 1 \text{ to } 2{,}790, \quad [4]$$

where P_{ijk} is the ijth element of the transition matrix resulting from making decision k. The P_{ijk} were based on the transition probabilities obtained from commercial records described later. The model accounted for animals moving from one state to a successive potential state determined by the law of probabilities contained in the transition matrices of probabilities of pregnancy, mortality, involuntary culling, and abortion rates defined by PAR, MIL, and PREG.

Net Revenue when Keeping the Cow

The NR_{i1} (net revenue for decision 1 = keep the cow) was calculated as a function of 6 economic factors: the milk income over feed cost ($IOFC$), the income of a newborn calf (INB), the cost of a dead cow (CDC), the cost of involuntary culling (CIC), and the cost of insemination (AI). In addition, a function of the cost of manure disposal and the value of nutrients excreted was included in the net revenue calculation ($EnvFactor$):

$$NR_{i1} = IOFC_i + INB_i - CDC_i - CIC_i - AI_i + EnvFactor_i \text{ for } i = 1 \text{ to } 2{,}790. \quad [5]$$

The milk income over feed cost ($IOFC$) was calculated as the milk value (Mv) less the feed cost (Fc). The milk value (Mv) was the product of milk production (MP) by milk price (Mp). The feed cost (Fc) was calculated as the value of the DMI, which was a function of the diet, PAR, MIL, and PREG. The diet was defined as the proportion of feed ingredients of alfalfa silage (F), high-moisture ear corn (C), and soybean meal (SBM) that are described later as part of the experimental design.

$$IOFC_{i1} = Mv_i - Fc_i = MP_i \times Mp - DMI_i \times (F\% \times Fp + C\% \times Cp + SBM\% \times SBMp) \text{ for } i = 1 \text{ to } 2{,}790. \quad [6]$$

The income of a newborn (INB) was the value of a newborn calf as defined by the economic value of a newborn male or female and their respective probabilities. The probability of a female newborn was set at 46.7% (Silva del Rio et al., 2007). The income of a newborn was realized during the 9th month of pregnancy (PREG = 9):

$$INB_{i1} = 0.467 \times FCp + (1 - 0.467) \times MCp \text{ for } i = 1 \text{ to } 2{,}790 \text{ and PREG} = 9, \quad [7]$$

where FCp is the price of a female calf and MCp is the price of a male calf.

The cost of disposal of a dead cow (CDC) was assessed as the composite cost of disposal and the cost of bringing a replacement. These costs were partially offset by the value of a newborn:

$$CDC_{i1} = Mr_i \times (Dc + HRc - INB_i) \quad \text{for } i = 1 \text{ to } 2{,}790, \qquad [8]$$

where Mr is the mortality rate, Dc is the disposal cost, and HRc is the heifer replacement cost.

The cost of involuntary culling (CIC) was assessed as the probability of involuntary culling (ICr) multiplied by the difference between heifer replacement cost and the value of a newborn less the salvage value (Sv) realized when culling a cow:

$$CIC_{i1} = ICr \times (HRc - INB_i - Sv) \quad \text{for } i = 1 \text{ to } 2{,}790. \qquad [9]$$

The cost of artificial insemination (AI) was calculated as the monthly estimated cost of a common reproductive program using AI including labor, semen, and pregnancy diagnosis and was estimated at \$20/mo (AgFA Wisconsin 2008, http://cdp.wisc.edu/AgFA.htm; accessed Mar. 3, 2009), which was charged to open cows in reproductive status (MIL ≥2 and PREG = 0).

The model included an environmental function applied to the manure and nutrient balance of the dairy herd system. The nutrient excreted ($NutValue$) that could be used for crop production less the cost of manure disposal (CMD) was included in the model. The cost of manure disposal was defined as a function of loading, transporting, unloading, and incorporating the excreted manure in nearby crop fields (Hadrich et al., 2008).

$$EnvFactor_{i1} = NutValue_i - CMD_i \quad \text{for } i = 1 \text{ to } 2{,}790 \qquad [10]$$

The cost of manure disposal was calculated to be \$16.50/mo per cow and was charged equally to all cows, assuming that the farm had a manure-storage facility that could hold manure for at least 6 mo and assuming that available fields to apply the manure were within an 8-km radius (Hadrich et al., 2008) and that the farm has cropland available to apply all produced manure. The nutrient excreted ($NutValue$) was calculated as a function of the N content of the manure. Nitrogen excreted was calculated as the difference of N ingested ($\text{DMI} \times \text{CP}/6.25$) and the N exported in the milk as the milk protein ($MP_i \times \text{MilkProtein}/6.38$) (DePeters and Cant, 1992). Both N ingested and N excreted were defined as part of the experimental design of the studied diets (see later discussion). The value of N was calculated as its value as fertilizer in crop fields assumed to be equivalent to the value of N content of urea: $\text{UreaValue} \times 46\%$ N content. This value was then multiplied by a factor to account for the value of other nutrients available in the excreted manure such as P, K, and microelements. The multiplication factor was set at 3, assuming that the full value of manure is 3 times the value of N as fertilizer (Pennington et al., 2009).

Net Revenue when Replacing the Cow

The net revenue for decision 2 (replace the cow) considered the voluntary replacement of a cow by a pregnant springer that enters the herd just before calving. It then calved and became cow in PAR = 1, MIL = 1, and PREG = 0. Immediate replacement to maintain the herd population is a standard assumption in DP (De Vries, 2006). As previously stated, DP requires reaching a steady state of the herd population dynamics to reach an optimal solution, and this is achieved when the population remains stable, which also reflects practical farm operation. The net revenue for replacing a cow was calculated as the difference of the salvage value of the cow leaving the herd less the difference of the heifer replacement cost (HRc) and the income of a newborn (INB):

$$NR_{i2} = Sv - (HRc - INB) \text{ for } i = 1 \text{ to } 2{,}790. \qquad [11]$$

Computer Model

The problem was set as a large Excel spreadsheet (5,580 columns and 2,790 rows). The problem was solved using the Risk Solver Platform with the Standard LP/Quadratic Engine.

Reproduction and Abortion Rates

Monthly pregnancy rates according to the dimensions of the model were obtained from a Midwest DHIA program that included 326,000 Holstein lactations during a 5-yr period (2003–2007) [AgSource DHI Cooperative Services (http://agsource.crinet.com/page249/DHI) provided the baseline data used to parameterize reproduction rates, involuntary culling, and mortality rates on Holstein cows]. Records included the actual number of cows becoming pregnant from one month to the next. Consequently, the probability of pregnancy occurring during a particular MIL was calculated by dividing the number of cows becoming pregnant during a month by the number of eligible cows the previous month. These values then represented the product of conception rates by service rates in every month. Table 1 shows primiparous and multiparous cows' pregnancy rates that were used in the model.

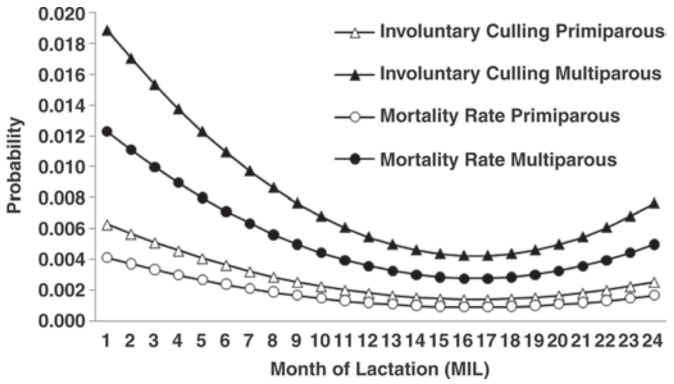

Figure 2. Mortality rate and involuntary culling in Midwest Holsteins. Involuntary culling is assumed to be 3 times the mortality rate. Source: Adapted from 326,000 Holstein lactations (2003–2007) provided by AgSource DHI Cooperative Services.

Abortion rates were not available in the DHIA database. These were obtained from De Vries (2006), who indicated a probability of abortion by month of gestation (2 to 8) of 3.5, 2.5, 1.5, 0.5, 0.25, 0.1, and 0.1%, respectively, with a total probability of abortion of 8.45% during a pregnancy. The probability of abortion in the first month was set at 0% because fetal embryonic loss during the first month was considered as if the cows did not become pregnant.

Mortality and Involuntary Culling Rates

Monthly mortality rates according to the dimensions of the model were obtained from the AgSource DHI Cooperative Services program that included 326,000 Holstein lactations during a 5-yr period (2003–2007). The database included records of number of cows that died in a particular month, which were used to calculate the mortality rate (*Mr*) by MIL and PAR. The database, however, did not report involuntary culling. Involuntary culling (*ICr*) was calculated as a function of mortality rate. Based on AgSource benchmark data for Holsteins (AgSource Cooperative Services, 2009), it was safe to assume the involuntary culling to be 3 times greater than the mortality rate (Figure 2). As expected, mortality and involuntary culling were greater in early lactation (right after calving), decreased toward mid-lactation, and increased again toward the end of lactation. Multiparous cows have greater mortality and involuntary culling than do primiparous cows.

Economic Factors

Average market conditions observed for Wisconsin in 2008 were used as the baseline. Milk price (*Mp*) was set at \$0.44/kg (\$18.92/100 lb) (Understanding Dairy Markets Web site, accessed March 10, 2009, http://future.aae.wisc.edu), the heifer replacement cost (*HRc*) at \$2,000 (Wisconsin USDA Agricultural Marketing Service Reports, 2008), the meat value at \$1.16/kg (Understanding Dairy Markets Web site, accessed March 10, 2009, http://future.aae.wisc.edu), and the salvage value (*Sv*) of a 726-kg culled Holstein cow at \$840.32. The disposal cost of a dead cow (*Dc*) including labor and machinery was estimated at \$100. The price of a female calf (*FCp*) was set at \$500 and the price of a male calf (*MCp*) at \$50 (Wisconsin USDA Agricultural Marketing Service Reports, 2008). The costs for feed ingredients was set at (\$/kg) 0.115 for alfalfa silage (calculated from alfalfa hay), 0.187 for high-moisture ear corn, and 0.366 for soybean meal (Understanding Dairy Markets Web site, accessed March 10, 2009, http://future.aae.wisc.edu). The value of urea was set at \$0.6071/kg (USDA Economic Research Service, 2008).

Experimental Design

Five diet treatments were studied. The model was solved for each one of the diets under different price scenarios to study the sensitivity of the outcomes to market conditions. The measured outcomes included the herd population structure, the replacement policy, the net revenue, and the amount of N excreted under each diet treatment. A suboptimal policy was also studied that included an imposed maximum N excretion of 12 kg/cow per month. A level of maximum N excretion of 12 kg/cow per month was empirically found by testing the model outputs under different scenarios.

Table 1. Pregnancy rate by MIL[1] and parity for Wisconsin Holsteins[2]

	Pregnancy rate (%)	
MIL	Primiparous	Multiparous
2	23.68	21.03
3	18.88	17.95
4	13.63	14.29
5	9.95	11.07
6	7.48	8.75
7	5.73	6.80
8	4.49	5.35
9	4.28	5.27
10	4.58	5.46
11	4.93	5.16
12	4.98	5.41
13	5.17	5.57
14	5.05	5.75
15	5.01	5.49

[1] MIL = month in lactation.

[2] Adapted from 326,000 Holstein lactations (2003–2007) provided by AgSource DHI Cooperative Services.

Dietary Treatments

Milk production, milk protein, and DMI for entire Holstein lactations in response to diets defined in proportions of alfalfa silage, high-moisture ear corn, and soybean meal to match every state defined in the model were based on a large controlled study in Wisconsin (Tessmann et al., 1991). No other study that could accommodate the objectives of this research was found in the literature.

Data from Tessmann et al. (1991) reported in weeks of lactation were aggregated at the monthly level to be integrated in the model created in this study. Following Tessmann et al. (1991), lactation was divided into 3 categories: early (1–3 mo), mid (4–7 mo), and late (8–22 mo). The proportion of alfalfa silage, high-moisture ear corn, and soybean meal varied for each diet in each lactation stage, as shown in Table 2. As in Tessmann et al. (1991), diets were isonitrogenous balanced at 19% CP in early and 17% CP in mid and late lactation, and dry cows received 11.4 (primiparous) and 15.9 (multiparous) kg of DMI/d with 17% CP. For each formulation, 2% of vitamins and mineral supplements were assumed to complete the diets, which were equal for all diets and were not included in the cost function.

Tessmann et al. (1991) reported results to only 44 weeks in lactation because it is usual to have a large proportion of the cows finishing their lactation around 11 mo after calving. However, the model structure required data beyond these months. To complete these data with the model structure, individual trend lines were fitted for wk 45 and later. A persistence factor was individually calculated for each lactation curve between the week in which a peak was reached and wk 44. The persistence factor was used to complete curves until MIL = 24. The DMI and milk protein were similarly adjusted after 44 wk in lactation to represent a similar function and trend as the milk production. The implications of this assumption are not critical in the model results because more than 90% of the herd population is contained within MIL $\leq$11. Therefore, only less than 10% of the herd population is affected by the extrapolation. Furthermore, an additional 8% of the herd population is between MIL 12 and 15, where the extrapolated points are near the last points of the original data.

Table 2. Percentage of ingredients on a dry basis in diets according to lactation stage[1,2]

Item	Month in lactation 1–3	4–7	8–22
Diet 1 (60% concentrate)			
Alfalfa silage	38	48	68
High-moisture ear corn	42	40	25
Soybean meal	18	10	5
Diet 2 (50% concentrate)			
Alfalfa silage	48	58	78
High-moisture ear corn	34	33	17
Soybean meal	16	7	3
Diet 3 (40% concentrate)			
Alfalfa silage	58	68	88
High-moisture ear corn	27	25	9
Soybean meal	13	5	1
Diet 4 (30% concentrate)			
Alfalfa silage	68	88	98
High-moisture ear corn	19	9	0
Soybean meal	11	1	0
Diet 5 (All-forage diet)			
Alfalfa silage	98	98	98
High-moisture ear corn	0	0	0
Soybean meal	0	0	0

[1] All diets had a 2% content of minerals and vitamins.

[2] Adapted from Tessmann et al. (1991).

Primiparous cows had flatter and more persistent milk lactation curves, whereas multiparous cows had less persistent lactation curves with higher peaks. Lactation peaks occurred between MIL 2 and 3, after which milk production decreased toward the end of lactation. The DMI followed the trend of milk production: increased in early lactation and decreased in late lactation. Milk protein, which decreased in early lactation and then increased steadily toward the end of lactation, was similar between primiparous and multiparous cows, although it had a greater response to diet composition in primiparous cows. The all-forage diet (diet 5), which had 98% alfalfa silage composition throughout the lactation, showed substantially lesser milk production in early and mid lactation than did the other diets for primiparous and multiparous cows. The diet with the greatest concentrate (diet 1) showed overall greater milk production in early lactation, although it was very comparable with the second-greatest concentrate diet (diet 2) for multiparous cows in mid and late lactation. For more details about the original study, please refer to Tessmann et al. (1991).

RESULTS AND DISCUSSION

Optimal Net Income Policy

The optimal replacement policy with favorable market conditions consistently called for the replacement of cows that were open at a certain MIL, depending on market conditions, parity, and diet. With only small variations, the replacement policy was similar for diets containing concentrate (diets 1 to 4). The optimal policy for 2008 market conditions and diets 1 to 4 called for the replacement of open primiparous cows on MIL 11 and multiparous open cows on MIL 10. With the all-forage diet (diet 5), the optimal policy called for

Table 3. Optimal policy, N excreted, and net revenue selected for model according to market conditions, diet, and N constraint

Market and constraint conditions	Diet[1]	MIL replacement[2]	N excretion (kg/cow per mo)	Net revenue ($/cow per mo)
2008 Favorable, milk $0.40/kg, corn $0.19/kg, replacement $2,000, no N constraint	1	11	12.56	132.16
	2	11	12.47	131.79
	3	11	12.55	116.92
	4	11	12.09	105.49
	5	12	11.35	79.84
2008 Unfavorable, milk $0.22/kg, corn $0.24/kg, replacement $1,500, no N constraint	1	9	12.38	15.06
	2	9	12.35	21.04
	3	9	12.46	18.71
	4	9	11.99	21.97
	5	10	11.18	18.38
2008 Favorable, milk $0.40/kg, corn $0.19/kg, replacement $2,000, N ≤12 kg/mo constraint	1	9[3]	12.00	119.84
	2	9[3]	12.00	126.36
	3	9[3]	12.00	104.86
	4	10	12.00	104.94
	5	12	11.35	79.84
2008 Unfavorable, milk $0.22/kg, corn $0.24/kg, replacement $1,500, N ≤12 kg/mo constraint	1	7[3]	12.00	10.98
	2	9[3]	12.00	19.88
	3	8[3]	12.00	14.84
	4	9	11.99	21.97
	5	10	11.18	18.38

[1] Diet 1 had 60% concentrate, diet 2 had 50% concentrate, diet 3 had 40% concentrate, diet 4 had 30% concentrate, and diet 5 was an all-forage diet. More detailed information is provided in Table 2.

[2] Suggested month in lactation (MIL) in which to replace a primiparous open cow; for all other cases, the replacement for multiparous cows occurred 1 MIL less.

[3] Herd distribution did not follow a defined pattern after the first parity.

the replacement of open primiparous cows on MIL 12 and multiparous open cows on MIL 11 (Table 3). When there was an unfavorable market defined with a low milk price ($0.22/kg), high corn price ($0.24/kg), and low replacement cost ($1,500), the policy for diets 1 to 4 called for the replacement of primiparous open cows on MIL 9 and multiparous open cows on MIL 8. For diet 5, the replacement policy called for the replacement of open cows on MIL 10 whether primiparous or multiparous. Along with the replacement policy, the model selected the maximum number of reproductive services to optimize farm net revenue. The last month in which reproductive services should be attempted was a month before the replacement month. For example, if the policy called for the replacement of open cows on MIL 11, then reproductive services should have been provided only until MIL 10.

In agreement with other studies (e.g., Groenendaal et al., 2004; De Vries, 2004, 2006), farmers' practices, and logical reasoning, it was not an economical decision to voluntarily replace pregnant cows. The future net revenue realized from a pregnant cow (the reward of newborn and milk production from future lactations) was always greater than the potential benefit realized by a potential replacement. However, if a cow reached a certain MIL without becoming pregnant, the future net revenue realized from that cow's replacement would exceed the potential net revenue derived from a pregnancy at that point in time or later. The optimal replacement time measured as the MIL of voluntary culling is a critical decision that can be found with the model. Whereas replacement due to mortality and involuntary culling occurs with little or no action of the manager, replacement from voluntary culling requires a thorough evaluation comparing the actual cow with a potential replacement in the long term, which includes several lactations in the future and potential replacements.

The modeling results indicated that primiparous cows should be given more chances at getting pregnant than should multiparous cows. This result is in agreement with previous reports (Dekkers et al., 1998; Groenendaal et al., 2004; De Vries, 2006). It was recommended that older cows receive one less breeding and be culled 1 mo earlier in lactation because their milk production was declining at a faster rate than the rate of primiparous cows. This might also be because older cows are at greater risk of involuntary culling.

Diet 5 held cows longer than diets 1 to 4. One reason for this result was that cows consuming the all-forage diet, although less productive, had markedly more persistent lactation curves than did cows on the other diets, which consequently resulted in greater monthly net revenues when keeping cows longer. Diets containing concentrates (diets 1 to 4) had similarly shaped lactation curves among them, although the diet with the greatest concentrate (diet 1) had the greatest net revenue of all diets.

It can be speculated that intensive feeding systems (i.e., using high concentrate levels in the diets, diets 1 to 4) could lead to a greater net revenue earlier in the lactation because of a high milk response to concentrates (Earleywine, 2001), which consequently would justify earlier replacement policies to have more cows close to the peak of lactation. In contrast, under an all-forage diet (diet 5), there would not be that much pressure for intense replacement policies earlier in lactation because of a lesser response of milk production to lactation stage and a greater persistency. Results of the model showed that keeping a cow longer with diet 5 would bring more benefits than replacing a cow.

Favorable market conditions such as those experienced in 2008 (high milk price, intermediate corn cost, and high replacement cost) called for lesser replacement rates than did unfavorable market conditions (low milk price, high corn price, and low replacement cost) (Table 3). Under these defined unfavorable market conditions, the model tries to allocate most of the cows to the peak of the lactation curve (where the milk income over feed cost increases) by increasing the replacement rate, which is supported by the fact that the replacement cost is relatively less. In the unlikely situation that the unfavorable market conditions of low milk price and high corn price are combined with a high replacement cost (e.g., \$2,000), the model would suggest keeping cows longer (up to 15 MIL, data not shown) than with the defined 2008 favorable market conditions.

Herd Structure

Table 4 is a steady-state Markovian representation of second-parity cows for 2008 market conditions fed diet 1 that maximizes dairy herd net revenue. The model finds the steady state of the herd as the equilibrium of cows entering the herd and cows leaving the herd, taking into account the transition probabilities defined as the probabilities of a cow becoming pregnant, being involuntarily culled, dying, or being voluntarily culled. Voluntary culling was a decision of the optimization model. The sum of all the coefficients in Table 4 (0.231) represents the proportion of cows of the whole herd standing in PAR = 2. Table 4 indicates that the proportion of cows starting second parity is 0.024, which is equivalent to saying that the category PAR = 2, MIL = 1, and PREG = 0 will have in equilibrium 2.4% of the herd (Figure 3). The proportion of cows finishing PAR = 2 is 0.012 (1.2% of the herd), which is the sum of the coefficients in the last column in Table 4. Consequently, the difference between these 2 numbers (0.024 − 0.012 = 0.012) indicates the proportion of cows that were voluntarily or involuntarily culled during PAR = 2. The proportion of voluntarily culled cows in PAR = 2 (decided by the model) was 0.0084 cows, which is found as the difference between open cows (PREG = 0) in MIL = 9 and the proportion of cows becoming pregnant (PREG = 1) in MIL = 10 (0.008862 − 0.000465). For each solution, there were 15 tables similar to Table 4, one per parity as part of the solution of the model.

The optimal proportion of the herd population for diets 1 to 4 was similar among these diets: 0.496, 0.243, 0.117, 0.057, and 0.053 for parity 1, 2, 3, 4, and 5 to 15, respectively (Figure 3a). As expected, the majority (85.7%) of the population was contained in the first 3 parities, and only a small fraction of animals reached later parities. Only 2.8% of animals would be in parity 5, and the proportion of cows reaching parity 10 or greater could be considered negligible. For the all-forage diet (diet 5), the optimal structure was 0.482, 0.244, 0.122, 0.061, and 0.061 for parity 1, 2, 3, 4, and 5 to 15, respectively (data not shown). Again, the majority (84.8%) of the population was contained in the first 3 parities. Only 3.1% of animals would be in parity 5, and less than 0.2% of them would reach parity 10.

As expected, the proportion of the herd population in increasing MIL categories decreased because of mortality and involuntary and voluntary culling (Figure 3b). Similar results as those illustrated in Figure 3b were found for the other diets containing concentrates [diets 2, 3, and 4 (data not shown)]. About 9.1% of the herd population was in the first MIL, whereas only 0.1% was in MIL = 19. No cows reached MIL ≥20. The decrease in proportion of animals followed a linear trend for MIL = 1 to 10 but fell sharply for MIL >10 because the optimal voluntary replacement policy calls for the replacement of open cows in MIL = 11 (PAR = 1) and in MIL = 10 (PAR ≥2). For diet 5, the sudden decline started 1 mo later.

It is important to indicate that the reported results show that the model was large enough to accommodate all potential cow states. Dimensioning the model for only 12 parities would probably be enough to accommodate a typical Holstein herd population. Knowing that the model only selects to replace open cows, the dimensions of the model could also be substantially reduced by giving the probability of replacement only to open states.

Net Revenue

The net revenue profile varied with several factors as stated in Equations 5 and 11. The single most important factor influencing net revenue was the milk income over feed cost (Equation 6), which was heavily affected by the diet ingredient composition and the milk price. For 2008 market conditions, the net revenue per month for diet 1 ranged between \$361 (PAR = 1, MIL = 2,

Table 4. Herd structure associated with maximum net revenue for second parity, high-concentrate diet, and 2008 market conditions[1]

	Pregnancy status[2]								
MIL	0	1	2	3	4	5	6	7	8
1	0.023785[3]								
2	0.023188								
3	0.017862	0.004801							
4	0.014336	0.003161	0.004703						
5	0.012213	0.002026	0.003103	0.004452					
6	0.010896	0.001338	0.001992	0.002944	0.004268				
7	0.009999	0.000946	0.001318	0.001894	0.002828	0.004142			
8	0.009351	0.000674	0.000934	0.001255	0.001822	0.002749	0.004068		
9	0.008862	0.000497	0.000667	0.000890	0.001209	0.001773	0.002703	0.004011	
10		0.000465	0.000492	0.000637	0.000859	0.001179	0.001747	0.002669	0.003966
11			0.000461	0.000470	0.000615	0.000838	0.001162	0.001726	0.002642
12				0.000441	0.000455	0.000601	0.000827	0.001150	0.001711
13					0.000427	0.000444	0.000593	0.000819	0.001140
14						0.000417	0.000439	0.000588	0.000813
15							0.000413	0.000435	0.000584
16								0.000409	0.000432
17									0.000407

[1] Numbers represent the proportion of cows of the entire herd in each specific state defined by parity, month in lactation (MIL), and pregnancy status when the model reaches steady state. No proportion of cows was found between MIL 18 and 24.

[2] 0 = open cows; 1 to 8 = months in gestation.

[3] Of the cows in the herd, 2.38% enter lactation 2. Similar herd structures were generated for each one of the 15 parities included in the model after each solution.

PREG = 0) and −\$152 (PAR ≥2, MIL = 23, PREG = 0), whereas for diet 5, net revenue per month ranged between \$332 (PAR = 1, MIL = 3, PREG = 1) and −\$152 (PAR ≥2, MIL = 23, PREG = 0).

The calculated net revenues followed a pattern similar to the lactation curves: first increased and reached a peak and then decreased to the end of lactation. In early to mid lactation, the greatest net revenues occurred when a cow was found pregnant or when the cow was in early pregnancy. Later in lactation, the greatest net revenues occurred at parturition when the revenue of a newborn was realized. The least net revenues in early and mid lactation occurred when the cows were open and in late lactation and in PREG = 8, the time when a cow is dry and not producing any milk revenues. The herd net revenues were the aggregation of all net revenues of all cows in a herd (proportion of cows by states) in a period of time of 1 mo. With the exception of diet 5, the concentrates of other diets varied in proportion of high-moisture ear corn, soybean meal, and alfalfa silage throughout the lactation as seen in Table 2. With diets 1 to 4, the herd was simultaneously fed 3 different diets, and consequently, the net revenue and N excretion were the aggregation of these 3 diets weighted by the proportion of cows in these corresponding states.

Maximum Net Revenue

The optimization of the model under the baseline scenario with 2008 market conditions (favorable) found that diet 1 had the maximum net revenue of \$132/mo per cow (Table 3). Diet 2 was only \$0.27/mo per cow less. The other diets fell substantially lower: diet 3, −\$15.24; diet 4, −\$26.67; and diet 5, −\$52.32. As a reference, De Vries (2004) reported net revenues varying between \$59 and −\$39/mo per cow. Although concentrate prices in 2008 were intermediate (not the cheapest nor the most expensive), diet 1, which had the greater contents of high-moisture ear corn and soybean meal, was the economic optimum in part because of the high response of milk to concentrates together with the high price of milk during 2008. Note that these comparisons were performed at optimal management conditions for each diet scenario.

Diet 3 (the third concentrate diet) behaved differently than the other diets; it did not always follow expected patterns between diets 2 and 4 with regard to net revenue and N excretion (Table 3). Although it is difficult to know exactly which factors and how the factors influenced these outcomes under the optimization framework, it is likely that the interaction among expected milk production, DMI and milk protein had the most influence on these results. On one side, the ratio of milk production over DMI determines dynamically the marginal income over feed cost, and on the other side, the relationship between milk production and milk protein determines dynamically the N excreted. Reviewing the original publication, Tessmann et al. (1991) reported no significant differences between diets 2, 3, and 4 for milk production and DMI. Interestingly, diet 3 had, however, a numerically greater DMI than diet

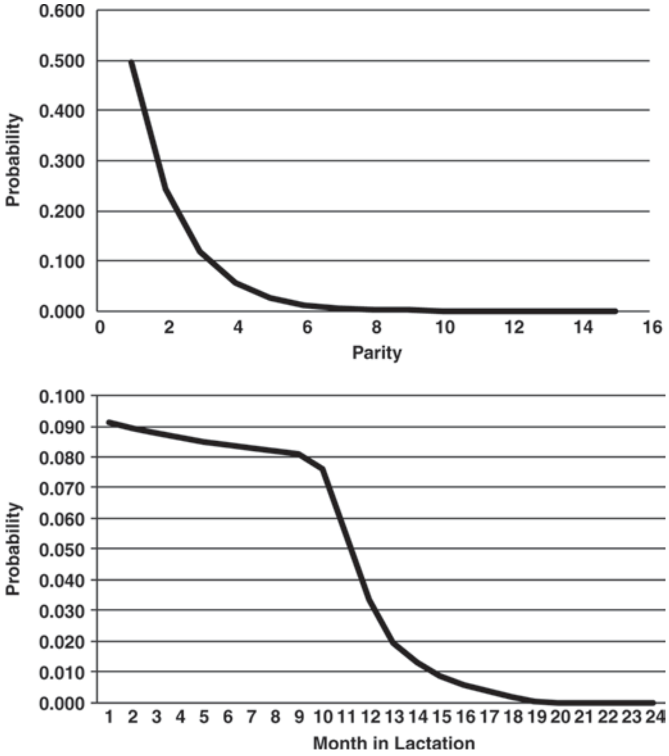

Figure 3. Proportion (probability) of herd population according to parity (top pane) and month in lactation (bottom pane) when the herd is in steady state for 2008 market conditions and fed the diet with the greatest concentrate (diet 1 in Table 2).

2 for primiparous cows. Examining the original data of DMI together with milk production, significantly lesser feed efficiency (milk/DMI) was found for diet 3 compared with diet 4 ($\alpha < 0.001$) for primiparous and multiparous cows. Consequently, under favorable market conditions that give a greater weight to milk value and less to feed value, the net revenue of diet 3 followed an expected pattern between diets 2 and 4. However, under unfavorable market conditions in which the cost of grain had a relatively greater weight, the net revenue of diet 3, which had a relatively greater DMI per unit of milk produced, was less than that of diet 4, when the opposite would have been expected.

With unfavorable market conditions that included a low milk price ($0.22/kg), high corn price ($0.24/kg), and low replacement cost ($1,500), net revenues for all diets decreased substantially. With these unfavorable price combinations, diet 4, which had a low level of concentrate and high forage content (Table 2), would have had the maximum net revenue of $22/mo per cow, followed by diet 2 ($21), and then diet 3 ($19). It is noteworthy that under unfavorable market conditions, diet 5 would have a greater net revenue than diet 1 ($18 vs. $15). Under these market conditions, diet 1 would have the least net revenue of all. As shown, the model can help in selecting the maximum net revenue diet according to market conditions. These results are consistent with previous analyses (Østergaard et al., 1996; Tedeschi et al., 2000; Earleywine, 2001) that have found that diet manipulation can have an important effect on farm net revenue according to lactations and market conditions.

Nitrogen Excretion

Substantial differences were found among scenarios regarding N excretion. In general, greater-concentrate diets were associated with greater N excretion (Mulligan et al., 2004). The model calculated the N excreted as the difference of N ingested and N exported with milk by the implied N efficiency utilization in milk production defined by the diets. Consequently, lesser N excretion is expected with diets with a greater conversion rate from fed N to milk protein. Although a small amount of ingested N is biologically used for cow body maintenance and fetus nutrition, these uses were ignored in the model. Therefore, estimates of N excreted might have been overestimated. However, these overestimates should only be minimal because of the fact that the N used for body maintenance and fetus nutrition is only a very small proportion of the N ingested and is commonly not included in similar analyses (e.g., Powell et al., 2008). In addition, N excretion was equally assessed with all the scenarios and all diets, so only minimal distortion would be expected because of this assumption among scenarios and diets.

Under optimal policies with market conditions for 2008, the least N excretion was found with diet 5 and was 11.35 kg of N/mo per cow (Table 3), and the maximum N excretion was found with diet 1 and was 12.56 kg of N/mo, a difference of 1.21 kg of N/mo or 14.52 kg of N/yr excreted per cow. The estimated N excretion (kg of N/mo per cow) for other diets was as follows: diet 2, 12.47; diet 3, 12.55; and diet 4, 12.09. Although N excreted was inversely associated with the level of concentrate in the diet and, consequently, with the level of DMI and economic outcome (for 2008 market conditions), diet 3 was an exception. Diet 3 had greater N excreted than did diet 2 and a level of N excreted very close to that of diet 1.

As with net revenues, diet 3 seemed to behave outside the patterns of contiguous diets. As previously discussed, the interaction of milk production and milk protein with diet 3 probably had the greater influence on these results. The original report from Tessmann et al. (1991) indicated that there were no significant differences among diets 2, 3, and 4 with respect to milk protein, and the milk protein content of diet 3 was in line in between diets 2 and 4. However, an analysis of the original data indicated that the difference between N ingested (calculated as a function of DMI and CP) and the N exported (calculated as a function of milk protein content and milk produced) was significantly greater for diet 3 than for diet 2 ($\alpha < 0.05$) for primiparous and multiparous cows. Consequently, calculated N excreted from diet 3 was always greater than that from diet 2 when it would have been expected to have been less. Under suboptimal conditions with a constraint limiting the amount of N excreted, diet 3 needed a greater population adjustment than did diet 2 to reach a herd structure that complied with such a restriction, increasing the selection of cow states that yielded less N excretion rather than selection of those states with greater net revenues.

The model could be used to help producers reach the maximum net revenue within the constraint of a maximum limit of N excretion to the environment based on herd structure and diet ingredient composition (Cabrera et al., 2006b). Depending on the number of cows on the farm, the cropland, and the environmental restrictions, fine-tuning of diets and herd structure to reach a goal of maximum N excretion could be critical. For example, if the nutrient management plan of the farm indicates that N excretion per cow should be not more than 12 kg/mo per cow, the model could be solved accordingly for this suboptimal condition.

With the maximum level of N excreted set at 12 kg of N/mo per cow and with the 2008 favorable market conditions (high milk price, intermediate corn price, and high replacement cost), the diets containing greater levels of concentrate complied with this restriction by drastically elevating the replacement rate (e.g., replacement suggested at 9 MIL instead of 11 MIL for diets 1 to 3), which evidently affected the net revenue (Table 3). Therefore, it would cost $12/mo per cow to reduce 0.56 kg of N excreted with diet 1, $5.43/mo per cow to reduce 0.47 kg of N excreted with diet 2, $12.06/mo per cow to reduce 0.55 kg of N excreted with diet 3, and $0.55/mo per cow to reduce 0.09 kg of N excreted with diet 4. Under the N excretion constraint, diet 2 yielded the greatest net revenue of $126 (Table 3). For diet 5, the maximum N excretion of 12 kg/mo per cow was irrelevant and did not alter the optimal replacement policy because the optimal net revenue was found at a level that was less than the N excretion limit imposed. With a 12 kg of N/mo maximum N excretion limit under unfavorable market conditions (low milk price, high corn price, and low replacement cost), diets 4 and 5 would retain their original solution, which would bring $22 and $18 with 11.99 and 11.18 kg of N/mo excreted, respectively. Diet 4 would have the best net revenue ($22), followed by diet 2 ($20) and then diet 5 ($18). Under unfavorable market conditions and an N excretion constraint, diet 1 would be the diet with the worst net return ($11; Table 3). Therefore, under unfavorable market conditions, it would cost $4/mo per cow to reduce 0.38 kg of N excreted with diet 1, $1/cow per mo to reduce 0.35 kg of N excreted with diet 2, and $4/mo per cow to reduce 0.46 kg of N excreted with diet 3. A restriction of 11.35 kg of N/mo excreted (level reached only with diet 5 under favorable market conditions) would result in nonfeasible solutions for all of the concentrate diets, meaning that none of them could reach an amount of N excretion that low.

CONCLUSIONS

A Markovian LP formulation and solution of the DP of dairy herd economic optimization problem represents a contribution to practical dairy herd decision-making tools applied to the replacement problem. It complements and adds to the value and policy interaction methods commonly used to solve large DP models. This study found the maximum net revenue for optimal and suboptimal dairy herd replacement policies for 5 different diets under different price scenarios. The model consistently found the following policies: (1) keep pregnant cows regardless of their production level (the net revenue to be realized with the newborn and subsequent lactations is always more valuable than a replacement); (2) allow primiparous cows to stay in the herd for more months—and try more services before culling—compared with their multiparous herd mates (because of the expectation of greater production in late lactation for the former compared with the latter); and (3) allow greater culling rates when economic market conditions determine low milk price, high corn price, and low replacement cost. Under favorable market conditions, diets with a high proportion of concentrates realize greater net revenues, but under unfavorable market conditions, diets with high forage content or with only alfalfa silage outperform high-concentrate diets. Diets with greater concentrates generated greater levels of N excreted. A suboptimal solution of the model to limit the N excretion per cow to 12 kg/mo resulted in the diet with the second-greatest level of concentrate (diet 2) providing the greatest net revenue under favorable market conditions. With unfavorable market conditions and under the same N excretion restriction, the least-concentrate content diet (diet 4) provided the greatest net revenue. The implementation of a Markovian linear program is an important advancement for dairy decision making that provides both robustness and versatility in operations research. The model could become a valuable tool to support economic decision making in dairy herd management.

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